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We analyze a simple model for the dynamics of a family system based on a video tape recording of the first interview between the family and a therapist. The model serves as a working example to show the possibility of abrupt transitions beyond a bifurcation point separating different behavioral domains. Such transitions separate one stable, steady state from another or a non-oscillatory state from periodic behavior. A modification of the basic model indicates how the passage through a bifurcation can be suppressed. Bifurcation phenomena sometimes accompanied by oscillations are known to abound in chemical and biological systems. Whereas bifurcations occur in the latter systems as a result of regulatory interactions at the molecular or cellular levels, the model suggests that similar bifurcations may occur on a more global level from interpersonal relations within the family. We compare the predictions of the model with the interventions of the therapist and discuss the relevance of the results to other patterns of family interactions which might also lead to cyclical behavior.

Introduction

Knowing the evolution of a system as a function of its environment and the interactions between its various elements is a major problem, be the system composed of molecules, cells, or populations of predators and preys. The same problem arises in a much more complex fashion in a family or in a larger social group. Obviously, the question of system evolution is linked to the problematics of change which is of primary importance in family therapy (Watzlawick, Weakland & Fisch, 1977).

Studies of physicochemical systems throw light on the various ways in which change in a system's behavior may be achieved (Nicolis & Prigogine, 1977). As will be discussed below, the evolution in response to a continuous change in some parameter value can be broadly classified into two types: (1) evolution towards a stable, steady state, which corresponds to homeostasis (Jackson, 1957) and illustrates the concept of equifinality (von Bertalanffy, 1968), i.e. evolution of a system to the same final state regardless of initial conditions; (2) evolution beyond a critical point of instability of a non-equilibrium, steady state. The latter phenomenon is associated with processes of self-organization which have been referred to by Prigogine under the general term of 'dissipative structures'

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(Prigogine, 1969; Nicolis & Prigogine, 1977; Prigogine & Stengers, 1984). A characteristic of evolutions of the second type is that they involve a discontinuous change in behavior when the steady state becomes unstable.

One of the prerequisites for the occurrence of instabilities in physicochemical systems is the existence of non-linearities in the equations which govern the system's evolution. Such non-linearities arise naturally in biology from the feedback processes which regulate the operation of living systems. We recover here the concept of 'deviation-amplifying feedback' introduced by Maruyama (1963) to account for the process of change in human systems. Further discussion of the application of this concept in human groups has been given by Wender (1968) and Hoffman (1971, 1981) who emphasized, in particular, the possibility of cyclic behavior resulting from interactions between members of a given family. Destabilizing interactions in human systems had previously been discussed by Bateson (1958, 1972), who referred to such processes of 'progressive escalation' as 'schismogenesis'.

As previously mentioned, dissipative structures arise beyond a critical point of instability called 'bifurcation point' in mathematics. The existence of such points, which separate two distinct modes of behavior, has been perceived intuitively by a number of workers, e.g. Rabkin (1976) who referred to them as points of crisis or 'saltus'.

Our goal in this paper is to relate the phenomena of self-organization beyond a bifurcation point in physicochemical and biological systems to the dynamics of family processes. We suggest, in particular, that oscillatory phenomena which are common in chemical (Winfree, 1974) and biological systems (see, e.g. Goldbeter & Caplan, 1976) may be paralleled by cyclic behavior in some family systems.

In the section 'Analysis of a family system', we present extracts from a video recorded at a first interview with a family. On the basis of this interview, we then analyze a series of simple models for this family system by the same methods utilized to study the time evolution of chemical, biological, or ecological systems. The analysis shows the possibility of abrupt transitions in behavior beyond a point of bifurcation, as well as the possibility of periodic behavior. We study several modifications of the model, one of which leads to the suppression of the bifurcation.

From the outset, let us stress that we do not wish to reduce the complex behavior of a family to that of a physicochemical system. In writing equations for the time evolution of a given family system, we are conscious that such equations encompass only a fraction of the processes which govern the system's evolution. As a counterbalance to this, we study a set of different models to determine how the form of the equations influences dynamic behavior. Whereas the very fact of writing down evolution equations may seem to reduce the richness of family processes by imprisoning them in a few mathematical relations we note, in contrast, that the analysis of these simple equations shows the possibility of multiple behavioral modes whose number is increased by the existence of bifurcations. Moreover, the occurrence of bifurcations closely depends on the particular interactions within the system. This further highlights the importance of the family *singularity* (Elkaïm, 1981, 1985).

Another result of this theoretical approach is that it lends support to the occurrence in family dynamics of cyclical processes and threshold phenomena whose existence has often been perceived by sheer intuition.

Analysis of a family system

The criteria for self-organization in open systems—appropriate non-linearity of evolution equations resulting from positive or negative regulatory processes—are probably satisfied in some (if not all) family systems. The goal of this work is to discuss, using a single, specific example, the possibility of bifurcations separating different behavioral domains in a family system.

The family considered is presented in the section 'Presentation of the family system' by means of extracts from a video tape made during the first interview with the therapist. Simple models for the evolution of this system are studied in the section 'Analysis of a two-variable model'.

Presentation of the family system

The therapeutical work described here belongs to the 'systemic' approach to family therapy (for general presentations, see Bowen, 1978; Hoffman, 1981; Minuchin, 1974; Selvini-Palazzoli *et al.*, 1978; Watzlawick *et al.*, 1967). This approach is characterized by a search for the function of the symptom within the family system rather than only in the patient. One of the functions of the symptom may be to maintain a certain form of homeostasis within the family (Jackson, 1957). The therapist tries to understand both the conditions for appearance and maintenance of the symptom, as well as its role in the dynamics of the family. His interventions aim at modifying the rules of the family organization in such a manner that the symptom ceases to be 'necessary' to the family.

In order to substantiate the process of model building, we give a brief presentation of the family and reproduce salient extracts from the first family therapy session.

The family comprises seven members, including five children: Léon (aged 12 1/2), Christian (aged 15), the identified patient Patrick (aged 25), his twin brother Jean, and Henriette (aged 27). The parents are in their late fifties. Jean and Henriette have left the family home and each is living on his/her own.

The problem presented concerns Patrick's behavior. He never goes out of the house, is active at night, and sleeps during the day. He drinks, inflicts injuries on himself, and is violent.

During the session, a coalition is soon formed between Henriette and Jean to support Patrick *vis-à-vis* their parents. Whilst claiming to be permissive, the father is described by the older children as authoritarian and inflexible. Throughout the early part of the session, the mother seems listless, protecting the two younger sons as best as she can and saying very little. There seem to be very few relations between the two members of the parental couple. The father says he regularly takes tranquilizers, the mother seems depressive and self-effacing.

As the session continues, one of the functions of the symptom begins to become increasingly evident to the therapist. Whenever, as a result of various factors such as the subject broached, one of the parents seems to find himself in difficulty, the identified patient behaves in a manner which, to the family, may appear aggressive. At that point, the subject of the conversation changes, the identified patient's behavior then becoming the focus of everybody's attention.

The following extracts (translated from the French) occur after a half-hour conversation. The therapist (M. Elkaïm) has already twice noted the process whereby—at moments when it is difficult to maintain the homeostasis of the family system—the identified patient distracts attention to himself.

Therapist (*Th*). Mrs X., do you agree with your husband when he says there are times when . . . he was tense, irritated and that, despite whatever he might do . . . he brought back home the problems he had at work?
Mr X. I've never brought my work problems home to her; that's just what I wanted to avoid, to bring these problems home . . . I've always kept them to myself.
Jean (*to his mother*). That's just it; that's one of the problems you often raised: your husband would come home at five o'clock and never have anything to say . . .
Mrs X. Yes, yes, that's right.
Th. Just a minute . . .
Patrick. With the problems, the way it came out was sickening (the father speaks at the same time); it was different, but whenever he lost his cool . . . he said: 'Yes, I'm not obliged to bring my money home. I could go and spend it on drink at the pub instead of bringing it home'.
Th. Patrick, if I may say so . . . What I find striking is that I put a question to mother and everybody talks but her. I know that generally mother doesn't say much at home . . . But let her at least speak now. What do you think, Mrs X.?
Mrs X. 'So he wasn't that bad-tempered', hmm . . . You could sense it—he didn't need to say anything. It was the way he spoke. He had this way of his. You could sense it . . . that something was wrong. In any case, we knew. He would tidy up the cupboards and he began to . . . I would say to myself: 'He's having his fit' . . . you get it in every house.
Th. I'm not saying that you don't get it in every house . . . But there were in fact times when your husband was under strain . . .
Mrs X. Yes, and it was . . . But I think he felt it physically, because you can feel it . . . (she weeps).
Th. That's right . . . But this is important . . . it's important. Something is obviously upsetting you and it weighed heavily on you and still does . . .
Mrs X. Oh, no! No, no . . .
Mr X. No, but it . . .
Th. Just a second, Mr X., allow me for just a second . . . But, Mrs X., what are you crying for then?
Mrs X. But at being here!
Th. Yes, and what does being here mean?
Mrs X. I don't know.
Th. It's important for me, Mrs X., that you should help me understand what makes you want to cry.
Mrs X. I always feel like crying . . .
Th. But this is important! How long has it been like this? (Jean moves.)
Mrs X. Since things went wrong.
Th. Since when have things been going wrong, Mrs X.?
Mrs X. I don't know, we didn't understand the children, so . . .
Patrick. Did you ever have a clue about anything . . .?
Mrs X. But that's just it . . .
Mr X. That's the sad thing about it . . .
Mrs X. We didn't do enough to . . .
Patrick. There's one thing you understood, you know very well and you've always hidden it from us, you're poor shits (shouting).
Mrs X. That's it, people would think . . .
Patrick. And then—what did you do to hide it from us? You used the dirtiest method!
Jean. Ours.
Patrick. Well . . . there's no point in blubbing! (The mother dries her tears and stops weeping.) (Everybody speaks at the same time.)
Patrick. Let her go and get stuffed.
Jean. I don't see why your mother should say she's a poor shit, even if you know she is. She doesn't need to tell you out loud!
Patrick. It's the way they went about it, precisely.
Jean. If you're aware of it . . .
Th. Patrick.
Henriette. Patrick, Patrick.
Th. I'm going to tell you something that will surprise you . . . I thing you are an exceptionally sensitive young man—protecting his parents . . . that's what I think and I'll explain why . . . When

your mother cries, it's very difficult for her, you then get all aggressive and so people focus their attention on you and her tears disappear right away, and then it starts all over again . . .

My intuition tells me—although I may be miles besides the point—that you are an extremely sensitive young man drawing towards yourself a whole series of the family's tensions, so that people can have a breather. What you're doing is to sacrifice yourself so that things can happen in such a way that people are protected and don't have to face up to their own problems.

My impression is that right now you are a nurse to your family and not at all the madman they treat you as.

You're the one who's extremely sensitive, who, without realizing it, through a series of acts and things he does, becomes the center of attention . . . which allows not paying attention to anything else that goes on and that is . . . your mother's constant sadness . . . and the tears she's always holding back.

Mrs X. Oh, mister, I'm not sad all the time!

Th. No. But you say you often feel like crying.

Mrs X. Yes, often . . .

Th. And father is somebody who feels comparatively dissatisfied at a number of things and I have the impression that without realizing it your problem is the way you have of helping and protecting them. I realize that in saying this I might seem to be the one who's mad . . . (laughter). Anyhow, that's the way I look at it.

(. . .)

I also think it important that mother and father should do a number of things as well and the thought occurs to me that a part of our task might consist in seeing you alone, Mr X., with your wife, and without the children, so that we can together think about how you could organize yourselves so as not to want to cry so often and to feel happier in what you are doing, so as to relieve Patrick who, as I see it, with his snarling ways and his shouting is helping you to divert your tension onto him. Are you nodding agreement, Henriette?

Henriette. I think you've hit on a mechanism which is rather true.

Th. You have that impression as well, Henriette?

Henriette. Yes.

Jean. Me too. Patrick has a tendency to think for others, to lay down final judgements for others. He always thinks in terms of the family. He doesn't seem to raise any problems for himself, he has always sorted everything out. The problems you raise (to Patrick) are always related to your father, your mother and your brothers. You raise a whole series of problems as regards the family and you always seem to exclude yourself, because as far as you yourself are concerned, you've solved everything, you've understood it all.

Patrick. I never said I'd solved anything.

Jean. It's the way you talk, the way you present problems.

Th. For me, what Jean is saying is that Patrick is so concerned about the family that he ends up by thinking of himself as unimportant.

Patrick. Not at all about the family. I say that what I'm going through now is on account of the family, of upbringing.

Th. I say there are two levels: the one at which you express yourself and what I see happening.

What you're expressing is an angry young man who does this (Th imitates Patrick making a threatening gesture) to his father. But what do I see? I see it happens whenever they're in danger. When mother starts crying, it comes out, and mother very soon stops crying and replies to her son, and mother can then breathe again. When father has a spot of difficulty, you rush in like that (imitating Patrick again). Father then begins to relax. What I'm saying is that it's on the cards that without realizing it you do your best to help them.

Thus, it's obviously very difficult to accept when you're told that the people you feel angry against are at the same time those you are trying to protect; I would like you to be able to think about yourself a little, to rest a bit and for the two of them to cope with their day-to-day problems, so that you can look after yourself as well!

After that, the therapist discusses with Patrick the things he likes: reading science fiction ('breaks in reality, things that don't quite look like what they are'), drinking ('completely different sensations, other perceptions').

During the discussion, Patrick's tone is very calm and the family listens attentively.

Th. Tell me, are you planning to spend your whole life like this with your parents, reading, dreaming . . . protecting them?
Patrick. Of course not.
Th. You can be a wonderful nurse like that, a nurse who reads science fiction at patients' bedsides.
Patrick. No, when you've reached 25, to become aware of certain things . . . perhaps it's too late to start again.
Th. It's interesting . . .
Henriette. You would like your parents to start again at 60 . . . (laughter).
Patrick. I didn't ask to start again, I ask . . . if they could understand . . . if they could leave me alone for Christ's sake!
Th. Patrick . . . Just a minute . . . I quite appreciate what you're saying . . . and I think that if you've got the impression that you should continue doing what you've been doing, well you should do so. You are right, Patrick . . . I mean for the time being your family can't manage without you . . . as you are now. I should in fact like you to continue to behave as you are doing, because it clearly has a function which is to help the family to keep going as it has been doing so far in an unbalanced way, with very great difficulty, although you are quite clearly unable to see how your parents could cope without you. That's my understanding of the situation . . . So I accept that what you tell me, and I think that as long as father and mother haven't changed they won't show that they can be happy otherwise than by being 'as dumb as suitcases'.† Maybe, as long as this won't be done, you will continue to feel obliged to protect them.

Analysis of a two-variable model

In this section, we analyze a simple model for the family system described in the previous section. The first problem encountered in the study of such a model concerns the number and the nature of the variables to be considered. The model must comprise a restricted number of variables if one wishes to obtain qualitative insights into its behavior, without resorting only to computer simulations. On the other hand, one should not neglect any essential interaction, i.e. any variable whose role is important for the dynamics of the system.

Retaining a balance between these tendencies, one should establish a hierarchy between the variables, according to their influence within the system. Thus, for the example studied it appears that the identified patient and his mother play a prominent role, followed by the father and by the other children of the couple. In fact, the therapist should also be considered as part of the system. As a first step, we shall consider a model based on the interactions between the mother and the identified patient.

The variables considered are, in fact, attributes of the members of the family system. These attributes are linked either to what is regarded by the family as the symptom itself (here, the aggressiveness' of the identified patient), or to the processes which participate in the production of the symptom and/or are affected by it (in the present case, the 'tension' of the parents).‡

The nature of the variables, their hierarchy, and the laws of interaction between them have been revealed by the analysis of the situation made by the therapist. Here, the therapist observes (see the section 'Presentation of the family system') that the aggressiveness of the son increases with the tension of the mother and, once initiated, abruptly reduces this tension. The system corresponding to these interactions is schematized in Figure 1.

The goal of our study is to determine how the variables A (aggressiveness of the son) and T (tension of the mother) evolve as a function of time. Mathematically, the time

† Literal translation of Patrick's own words.
‡ We are aware of the danger there is in utilizing terms such as 'aggressiveness' or 'tension'. These terms, of which we do not wish to approve implicitly the usual acceptation, recover multiple verbal and non-verbal manifestations which it would have been difficult to describe in detail.

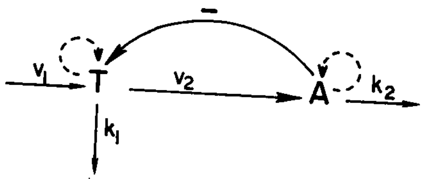

Fig. 1. Scheme of the family system. Arrows indicate production of the son's aggressiveness (A) *by the mother's tension* (T) *and inhibition* (–) *of* T *by* A. *The system is open: tension builds up at a constant rate* v_1, *whereas* T *and* A *are eliminated autonomously by mother and son with the rate constants* k_1 *and* k_2, *respectively. The dashed lines represent the autocatalytic effects exerted by* A *and* T *on their own production*

evolution of these variables is described by differential equations. In each equation, one can distinguish production terms, whose sign is positive, and destruction terms which possess a negative sign. These terms include the eventual regulations.

Once the variables and the modes of interaction are identified, a mathematical representation of these interactions remains to be chosen in order to obtain the differential equations describing the evolution of the system. It is out of the question to look for expressions yielding quantitative results in the study of a system as complex as a family. The multiplicity of choices open to any human system would not permit this. However, one can reasonably hope to obtain qualitative results by means of a phenomenological representation of interactions which occur in a repetitive (though not necessarily periodic) manner.

Evolution equations for the basic model

The two-variable model based on the observations made in the section 'Presentation of the family system' rests on the following hypotheses (see Fig. 1 for a scheme of the model). (1) The tension of the mother (variable T) builds up at a rate proportional to T. (2) This tension elicits the aggressiveness of the son (variable A) which builds up with a rate proportional to A. (3) The aggressiveness of the son inhibits the tension of the mother. (4) Both the aggressiveness of the son and the tension of the mother can be eliminated in an autonomous fashion, with a rate proportional to A and T, respectively.

In this model, the time evolution of variables A and T is governed by the differential equations:

$$\begin{aligned}\frac{\mathrm{d}T}{\mathrm{d}t} &= \frac{v_1 T}{\alpha_1 + A} - k_1 T \\ \frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{v_2 A T}{\alpha_2 + T} - k_2 A\end{aligned} \tag{1}$$

In the equation for T, the first term denotes a source of tension which is inhibited by the son's aggressiveness. Constant α_1 measures the value of A which yields 50% inhibition. When $A \ll \alpha_1$, there is no noticeable reduction of T by A, whereas the reduction becomes maximal when A exceeds α_1. The appearance of T in the numerator results from the hypothesis that the rate of increase in T at a given time is proportional to the magnitude of the tension at this moment (the effect of relaxing this hypothesis will be considered in the section 'Modifications of the model' below). The second term, $-k_1 T$, relates to the autonomous elimination of the tension by the mother regardless of the son's aggressiveness.

In the equation for A, the first term relates to the activation of A by T, with a constant α_2 denoting the value of T yielding 50% of maximal effect. When $T \ll \alpha_2$, no significant increase in A is noticeable, whereas the enhancement in A saturates at a constant maximum level when $T \gg \alpha_2$. The presence of A in the numerator expresses the hypothesis that the rate of increase in A at a given time is proportional to the value of A at this moment. Intuitively, one can comprehend this hypothesis by noticing that a violent person can augment his/her aggressiveness faster than if initially calm (the effect of relaxing this hypothesis will be analyzed in the section 'Modifications of the model' below). The second term, $-k_2A$, reflects the possibility for the son to eliminate aggressiveness in an autonomous manner. In the absence of such a term, aggressiveness would increase boundlessly in the course of time.

On the basis of the qualitative interactions sketched in Figure 1, one could write the various terms appearing in the differential Eqns (1) in more than a single way (see the 'Discussion' below). The mathematical form of the inhibition and activation functions in Eqns (1) is identical to that of functions governing activation and inhibition processes in biochemical kinetics.

Steady states

The simplest and most convenient way to obtain information on the dynamic behavior of the system governed by Eqns (1) is to determine the steady states admitted by these equations, as well as their stability properties. Let us recall that a steady state is a state in which the variables A and T remain constant in time. Mathematically, this condition is expressed by the relations $(\mathrm{d}T/\mathrm{d}t) = (\mathrm{d}A/\mathrm{d}t) = 0$. Application of such a condition to Eqns (1) shows that the system can admit two steady states. The first one is the *trivial* (i.e. nil) state

$$A = T = 0, \tag{2}$$

which exists for all values of the system's parameters.

There also exists a second, *non-trivial* steady state given by

$$A = \frac{v_1}{k_1} - \alpha_1, \qquad T = \frac{k_2\alpha_2}{v_2 - k_2}. \tag{3}$$

Since A and T can have only positive or zero values, the existence of the non-trivial steady state is subjected to the conditions

$$k_1 < \frac{v_1}{\alpha_1}, \qquad k_2 < v_2. \tag{4}$$

If conditions (4) are not satisfied, which happens for large values of k_1 and k_2, the only steady state admitted by the system will be the trivial state $A = T = 0$. A similar result holds when $k_2 < v_2$ and $k_1 > (v_1/\alpha_1)$. Besides, if $k_2 > v_2$ and $k_1 < (v_1/\alpha_1)$, A will go to zero, while T will increase with time.

A simple manner to determine the stability properties of a steady state is to apply to the system in such a state infinitesimal perturbations, i.e. to slightly displace the variables (here, A and T) from their steady state values. If the perturbations grow in time, the steady state is unstable. In contrast, this state is stable if the perturbations regress and the system returns to the steady state after a sufficiently long time. Application of this *linear stability analysis* to the system governed by Eqns (1) shows that the trivial, steady state $A = T = 0$ is stable as long as

$$k_1 > \frac{v_1}{\alpha_1}, \tag{5}$$

and becomes unstable when

$$k_1 < \frac{v_1}{\alpha_1}. \tag{6}$$

As to the non-trivial, steady state given by Eqns (3), the stability analysis shows that when it exists—i.e. when conditions (4) hold—this state is always marginally stable: perturbations around this state can neither grow nor regress, and the system undergoes periodic behavior (see below).

Transitions between behavioral modes beyond a bifurcation

The stability analysis performed for the two steady states admitted by the system governed by Eqns (1) reveals the existence of a bifurcation point that separates two qualitatively distinct modes of behavior. This transition beyond a critical parameter value is illustrated in Figure 2 which, for a given ratio (v_1/α_1), yields the schematic bifurcation diagram of Figure 3.

The parameter which is the most appropriate for studying the transition is constant k_1. Indeed conditions (4) indicate that the trivial state $A = T = 0$ is the only steady state asccessible to the system when k_1 is larger than the ratio (v_1/α_1). Moreover, as shown by stability analysis [see condition (5)] the trivial state is then always stable.

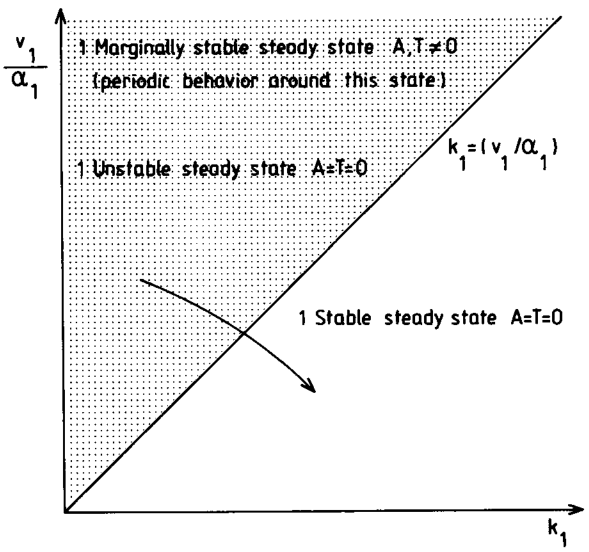

Fig. 2. Bifurcation diagram. The line $k_1 = (v_1/\alpha_1)$ *is the locus of points which separate two distinct behavioral domains. For a given value of* k_1*, an increase in* v_1 *or decrease in* α_1 *produces an abrupt switch from the trivial steady state to periodic behavior. Conversely, for a given ratio* (v_1/α_1)*, a similar switch occurs when decreasing* k_1*. The arrow represents some 'therapeutical path' that would bring the system to the state* $A = T = 0$ *across the bifurcation line. (See Goldbeter & Segel, 1980 for a biological example of similar transitions between different behavioral domains in parameter space)*

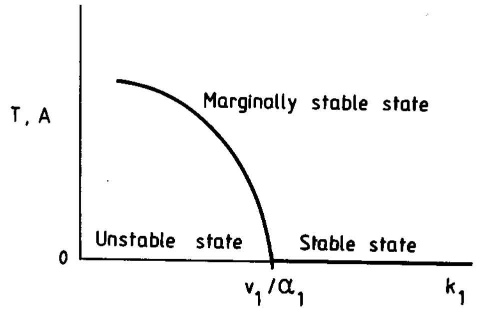

Fig. 3. Schematic bifurcation diagram illustrating the transition between distinct behavioral modes beyond a critical value of parameter k_1 in the system described by Eqns (1). The critical value $k_1 = (v_1/\alpha_1)$ corresponds to a bifurcation point. When k_1 exceeds this value, the system evolves towards the stable state $A = T = 0$. This state becomes unstable when k_1 drops below its critical value; a new, marginally stable steady state then appears, around which the system undergoes sustained oscillations (Fig. 5) corresponding to an infinity of closed curves in the phase plane (A, T) (Fig. 4)

On the other hand, the condition (6) for the instability of the trivial steady state coincides with one of the conditions (4) for the existence of the non-trivial steady state. The latter state is marginally stable, which result implies (see Nicolis & Prigogine, 1977) that in the phase plane (A, T), the non-trivial steady state is surrounded by an infinity of closed curves which represent the trajectories followed in the phase plane for different initial conditions (Fig. 4). These closed curves correspond to sustained oscillations of A and T as a function of time (Fig. 5). The trajectory followed by the system—and, hence, the amplitude and period of oscillation—depend on the initial values of A and T. In the absence of any new perturbation, the system will indefinitely pass through the point corresponding to the initial condition after completing a cycle.

Oscillations of the kind shown in Figures 4 and 5 are known in some ecological models under the name of Lotka-Volterra oscillations (Nicolis & Prigogine, 1977). This behavior has to be contrasted with limit cycle oscillations which correspond to a unique closed curve around an unstable steady state in the phase plane (see Goldbeter & Caplan, 1976 and Nicolis & Prigogine, 1977 for examples of chemical and biological systems admitting limit cycle oscillations).

An approximate value for the period τ of the oscillations in Figure 5 is given by the expression

$$\tau = 2\pi \sqrt{v_1 v_2}/\sqrt{k_1 k_2 (v_1 - k_1 \alpha_1)(v_2 - k_2)}. \tag{7}$$

The period is thus a function of most parameters of the system, which result emphasizes the fact that periodic behavior is a dynamic property of the family system as a whole, resulting from all interactions between the variables and the external world.

Modifications of the model

It is of interest to determine how slight modifications of the basic equations affect the dynamic behavior of the model. We shall examine in turn the effect of two such alterations.

First, if we suppose that the rate of increase of the son's aggressiveness is independent from the level of aggressiveness, then the equation for the evolution of A is replaced by

$$\frac{dA}{dt} = \frac{v_2 T}{\alpha_2 + T} - k_2 A, \tag{8}$$

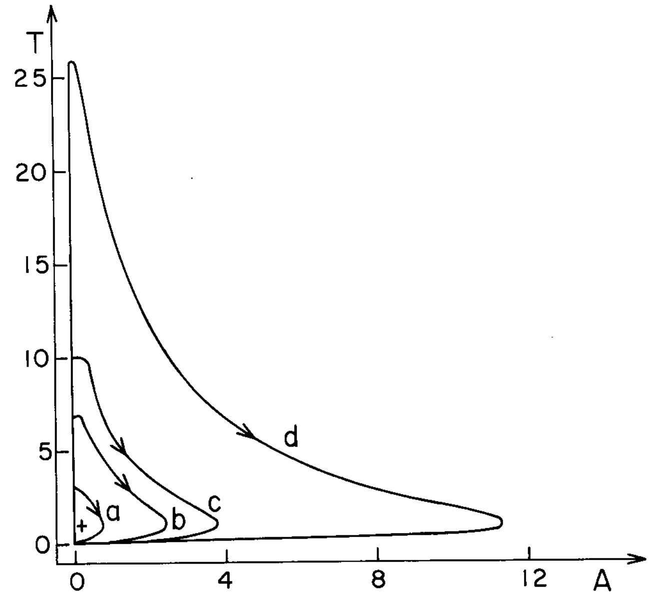

Fig. 4. Behavior in phase plane where mother's tension T *is plotted against aggressiveness* A *of the identified patient. The system can follow any of an infinity of closed trajectories around the marginally stable steady state (+). Curves a–d are obtained by numerical integration of Eqns (1) on a computer, for four different initial conditions. The (arbitrary) values of the parameters are* $v_1 = 1$, $v_2 = 2$, $k_1 = 0.9$, $k_2 = 1$, $\alpha_1 = \alpha_2 = 1$ (k_1 *is less than its critical value which is here equal to unity). The steady state corresponding to these parameter values is* $T = 1$, $A = 0.111$

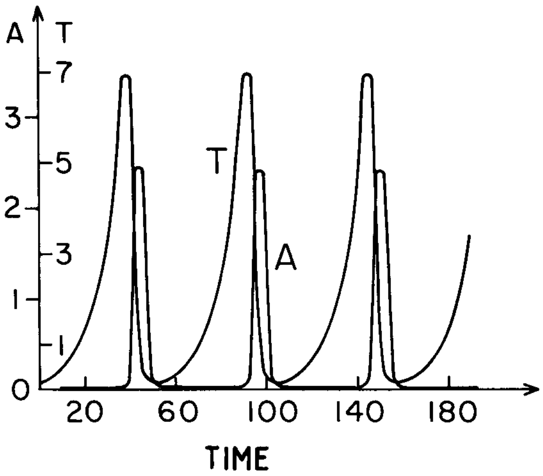

Fig. 5. Temporal oscillations of tension T *and aggressiveness* A. *The system undergoes a repetitive pattern of phases in which* T *rises, followed successively by a rise in* A, *a decrease in* T *and a decrease in* A. *The curves correspond to curve b of Figure 4. Time and variables are in arbitrary units*

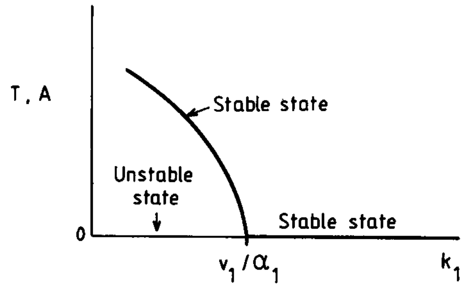

Fig. 6. Bifurcation diagram illustrating the transition between distinct stable steady states as a function of k_1 *when the model is modified according to Eqn (8)*

whereas the evolution equation for T remains given by Eqns (1).

The analysis of these equations shows that there still exists a trivial steady state $A = T = 0$ which is stable for $k_1 > (v_1/\alpha_1)$ and unstable for k_1 smaller than this critical value. As in the preceding model, a non-trivial steady state A, $T \neq 0$ exists provided k_1 and k_2 have sufficiently low values. The non-trivial steady state, when it exists, is always stable. These results lead to the bifurcation diagram depicted in Figure 6.

Only damped oscillations can be observed in this model, when the system evolves towards the stable non-trivial steady state from a given initial condition. The impossibility of sustained periodic behavior can be demonstrated *a priori* by means of a simple mathematical criterion applied to the evolution equations (see Elkaim, Goldbeter & Goldbeter-Merinfeld, 1980). This impossibility stems from the fact that the modified system of equations has a degree of non-linearity which is not large enough to permit the maintenance of a periodic regime.

Secondly, in addition to the above modification, let us now assume that the rate of increase in the mother's tension is independent of the level of T. Then, the evolution equation for T is given by

$$\frac{dT}{dt} = \frac{v_1}{\alpha_1 + A} - k_1 T, \tag{9}$$

with the equation for A given by Eqn (8). The trivial state $A = T = 0$ is no more a solution of the steady state equations. The evolution equations now admit only a single

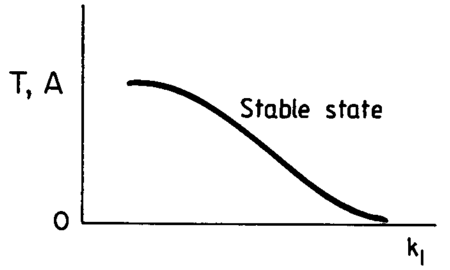

Fig. 7. Evolution of the model modified according to Eqns (8) and (9) as a function of parameter k_1. *Here, the system admits a unique steady state regardless of the value of* k_1 *and tends towards the state* $T = A = 0$ *when* k_1 *increases, without passing through a bifurcation*

non-trivial steady state. For increasing values of parameter k_1, A and T still tend toward zero but, this time, without going through a bifurcation (Fig. 7).†

Discussion

Within the family system considered, it appears that one of the essential interactions links the identified patient and his mother. The tension of the mother elicits her son's aggressiveness, whereas the latter inhibits the manifestations of maternal tension. We have studied a series of simple models based on these interactions by considering the time evolution of mother's tension (T) and son's aggressiveness (A).

In the model schematized in Figure 1, three parameters play a prominent role in controlling the system's evolution. These are: v_1, measuring the rate of increase in tension T; k_1, measuring the rate of autonomous elimination of tension T by the mother; α_1, which yields the value of A giving half-maximum inhibition of T by A (the larger α_1, the larger the value of A required to produce significant negative feedback of A over T).

We have seen that the critical value $k_1 = (v_1/\alpha_1)$ corresponds to a point of bifurcation (Figs 2 and 3). Indeed, when $k_1 > (v_1/\alpha_1)$, the system evolves towards a stable state in which T and A are nil. When $k_1 < (v_1/\alpha_1)$, the state $T = A = 0$ is unstable and can never be reached. Provided that $k_2 < v_2$, another steady state exists in these conditions. This state, in which A and T have values different from zero, is marginally stable. As a result, the system undergoes sustained oscillations around the marginally stable state, corresponding to a set of closed curves in the phase space (A, T) (Figs 4 and 5). *There is thus a discontinuous transition from a stable steady state* (A = T = *0*) *to a periodic regime* (*cycle* A, T).

Some modifications of the model lead to different conclusions. If the rate of increase of the son's aggressiveness at a given time is assumed to be independent of the level of aggressiveness at this moment, the system undergoes a discontinuous transition from a stable steady state ($A = T = 0$) to another, non-trivial steady state (A, $T \neq 0$) (Fig. 6).

If, in addition to the previous modification, the rate of increase in the mother's tension T is assumed to be independent of T, the bifurcation disappears and the system admits only a single, non-trivial steady state (A, $T \neq 0$) regardless of the value of parameter k_1 (Fig. 7). *The existence of a transition between behavioral domains beyond a bifurcation therefore depends not only on the interactions between variables* A *and* T, *but also on the feedback that each of these variables exerts on its own evolution.*

Link with the therapeutical interventions

The behavior of the model, as revealed from stability analysis, agrees with the therapeutical intervention (see the section 'Presentation of the family system') which consisted in helping the mother to better eliminate her tension in an autonomous manner (this corresponds, in the model, to an increase in parameter k_1). For a small value of k_1, only the non-zero steady state of A and T can be reached. This state is either periodic

† From the point of view of model-building, it may be of interest to note that the equations for A and T were first written in the form of (8) and (9). When preliminary inspection of these equations showed that they could never produce sustained periodicity, their degree of non-linearity was raised to allow for the possibility of such a mode of behavior. This was done by assuming that the rates of increase in A and T are proportional, respectively, to A and T, yielding the system of equations (1). The latter system was then showed to possess a bifurcation leading to periodic behavior.

(Figs 2 to 5)—which corresponds to repetitive, alternating phases of aggressiveness and tension—or stationary (Fig. 6). When constant k_1 increases beyond the value (v_1/α_1), the system switches abruptly to the state $A = T = 0$. From a therapeutic point of view, the latter situation is more satisfactory, since it represents a reduction in tension and aggressiveness. It should be noted that in a system which presents a weaker coupling and a decreased degree of non-linearity, the passage to the state $A = T = 0$ occurs in a continuous fashion, and only when k_1 tends to an infinite value (Fig. 7).

The question arises as to whether the use of k_1 as *bifurcation parameter* introduces any bias in the presentation of the results by conferring on k_1 a special status with respect to other system parameters. The answer to this is twofold. First, to have a steady state with $A, T \neq 0$, the conditions (4) require that both k_1 and k_2 have sufficiently small values with respect to v_1, α_1 and v_2. Once these conditions are satisfied, the only requirement for instability of the state $A = T = 0$ is $k_1 < (v_1/\alpha_1)$.

The condition of instability of the trivial steady state can therefore be expressed in terms of a critical value of either k_1, v_1, or α_1. For a given value of the other two parameters, the passage to the stable state $A = T = 0$ can thus be obtained (see arrow in Fig. 2) by either increasing k_1 (as discussed above), decreasing v_1 (i.e. decreasing the rate at which T produces A), or increasing α_1 (i.e. augmenting the range at which A significantly reduces T: this would occur, for example, if the mother continued to cry during the episode of the aggressiveness of the son . . .).

A second type of therapeutic intervention consists in an attempt to modify the rules governing the system's evolution. The *prescription of the symptom*: 'I would like you to continue to behave effectively as you behave' accompanied by a *positive reframing* of the symptom: 'Because, clearly it has a function which is to help the family to hold' and by a *paradoxical comment* which contradicts the official rules: 'The family is not capable, at this moment, to do without you . . . as you are at this moment' acts at the level of the feedback loop $T \circlearrowright A$. This prescription changes the evolution rules of the system and thereby permits the appearance of a new set of rules. The paradoxical comment has indeed described the function of the symptom in such a way that the latter can only be perpetuated by going against the anterior rules[†] (see also Watzlawick *et al.*, 1967; Selvini-Palazzoli *et al.*, 1978).

From the moment the prescription of the symptom accompanied by a paradoxical comment do not allow the system to use the feedback loop that it followed at the expense of other modes of interactions, enlarging the realm of available solutions becomes possible.

Situation and extension of the analysis

The model brings to light the possible occurrence of phenomena such as cyclical behavior and bifurcations, which were not perceived explicitly before being suggested by the analysis. In this sense, although the results concur with the views and prescriptions of the therapist, the model is more than just a mathematical formulation or metaphor of his conclusions. At the same time, however, the purpose of analyzing the model remains

[†] One may wonder how to reconcile a prescription of the symptom with a previous intervention where the therapist speaks of the necessity for the parents 'to change if they can and wish to change'. As a matter of fact, the therapist utilizes this first intervention to test the function of the symptom and the rigidity of the family rules. After the reaction of Patrick: 'It may be too late to start again', the therapist, by prescribing the symptom as long as the parents 'will not have evolved enough', creates the paradoxical comment. From this moment, the symptom reveals the rules that it was supposed to mask and thereby the evolution rules of the system can change.

conceptual rather than therapeutical: even if such an ambition were nurtured, the specific analysis would come too late to be used in the course of the therapy session.

Bifurcations and oscillatory phenomena occur in living systems at all levels of biological organization. To give but a few examples, they arise in biochemical systems from enzymatic or genetic regulation. In neurons they originate from the excitable properties of nerve membranes, whereas other rhythmic phenomena are associated with hormonal regulation. Our analysis suggests that above these 'lower level' bifurcations, there may be 'global' bifurcations arising here from interpersonal relations within a family. (At this higher level of description, we need not be concerned with the existence of bifurcations at lower systemic levels, e.g. in the firing of neurons: as the two levels of description are so far apart, dynamic phenomena on the two scales should not interfere.) Bifurcations probably occur on a macroscopic level in other social processes, e.g. in economy. A related example is provided in population dynamics by oscillations which occur as a result of predator–prey interactions.

When compared to the complexity of real family dynamics, the model analyzed here certainly appears as a gross simplification. We take it only as a working example to discuss the possibility of bifurcations and periodicity in some family systems. The equations governing the behavior of the model can be modified in several ways to give a more faithful picture. Increasing the non-linearity of the feedback loop $T \rightleftarrows A$ does not affect the results reported here (Elkaïm *et al.*, 1980). The source of tension could be made an explicit function of aggressiveness A. A relation more complicated than simple proportionality could be taken for the feedback of A and T on their respective sources.[†] Finally, the parameters of the system are certainly not constant in time (here is a further difference with chemical systems studied *in vitro*!). The differential equations describing the evolution of the family system therefore have a stochastic rather than a deterministic nature. As a consequence, the *periodic* behavior found in the present analysis more probably corresponds to a repetitive—though not strictly periodic—pattern of alternating bursts in tension and aggressiveness.

Generalization of the model

The present approach is based on the analysis of a video tape recording of a first interview with a family. After completing the analysis, it appeared to us that the model can be of more general significance. Many cases in clinical practice could be comprehended within a similar framework. As an example, a common situation of triangular interactions between a couple and a child is represented by the scheme of Figure 8. Here, F and M represent some measure of the tension between father and mother, respectively; S is some symptom of a child, which is elicited by the parental tension and exerts a negative feedback on F and M (this is the prototype of a well-known situation (Minuchin, 1974; Bowen, 1978; Haley, 1980) in which the 'problem' of a child plays an important role in maintaining family homeostasis). The analogy between Figures 1 and 8 is striking; some further situations that may lead to cyclic behavior are discussed by Wender (1968).

The periodicities discussed here would arise from interpersonal interactions. It is also possible that in particular cases (e.g. manic depression?), periodic behavior may largely originate from some underlying biochemical oscillation. The present analysis suggests,

[†] For example, the self-activations of A and T could saturate, i.e. reach a maximum value at large values of A and T. The absence of such saturating effects in the autocatalytic processes can lead to situations in which one or more of the variables takes infinite values: the system then ceases to be bounded and 'explodes'. Such a numerical accident, which does not happen in the two-variable model studied here, has occurred in a three-variable extension studied by Elkaïm *et al.* (1980). In order to avoid such 'runaways', saturating effects should be taken into account, as done for the interactions between variables A and T.

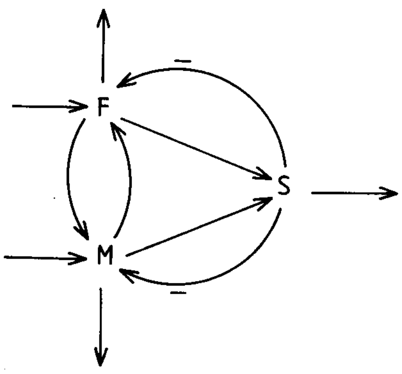

Fig. 8. Generalization of the model to triangular interactions in the family system. Here, F *and* M *characterize some tense relationship between the parents (*F *being the tension of the father and* M *that of the mother). A symptom* S *of one of the couple's children is linked to the parental tension and exerts an inhibitory effect on it. Each variable has a source (external or internal to the system) and can be eliminated in an autonomous manner*

more generally, that bifurcations leading to abrupt transitions between steady states or to periodic behavior may arise from a number of different patterns of interactions in family systems.

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